Atmospheric absorption by IR-sensitive molecules

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Introduction

In recent papers and blogs by Claes Johnson, here, here and here, it has been argued that what is called "back-radiation" is not a physical phenomenon. [**1, 2, 3**] Heat flows and radiates spontaneously from warm to cold. The fact that for some cases between two plates, *e.g.* the surface of the earth and the representation of the atmosphere by one single slab, the formulation for heat flow with the non-physical back-radiation gives the same temperature distribution as the formulation based on the correct interpretation of the law of Stefan-Boltzmann (SB), makes some people to believe that back-radiation exists and that heat can flow from cold to warm!
It will be shown in this paper that in the one slab model with the so-called two-stream formulation and thereby back-radiation occurs an absorption twice as high as compared to the one-stream formulation from warm to cold according to the Second Law of Thermodynamics.

Others are more prudent and forwarded the argument that the Second Law of Thermodynamics involves the "net" flux of heat from warm to cold!
During the development of the model of this paper attention was given to such remarks by always defining the heat flux between two surfaces from the warmer one to the colder one, according to a correct formulation of the law of SB. In this way we avoided the philosophical discussions, started more than 200 years ago between Prevost and Fourier, whether or not two black bodies always emit heat to each other as function of their temperatures, and in case of two black bodies with equal temperatures such heat fluxes, equal in magnitude but with opposite sign, result into a zero net flux of heat.

By introducing the one-stream formulation we had a model without back-radiation. The results were coherent, the same temperature distribution was found as for one-slab models with the two-stream formulation of heat flow with the so-called Prevost-Fourier type of heat sources.
We could live with the misnomer back-radiation but we could not accept the huge numbers given to it.
We will proof in this paper that there is only one-stream flow of heat from warm to cold and not two-stream flow of heat. Back-radiation does not exists, the two-stream model gives spurious absorptions, as will be shown analytically for a one slab model and for mulch-layer models.

Still others claim that they have measured back-radiation!
Claes Johnson, has argued that pyrgeometers do not measure heat fluxes but rather a signal related to temperature. [**4**] A micro-processor converts the signal by an algorithm based on SB to W/m^2 and it is called "back-radiation of heat" from the colder sky to the warmer surface of the planet!

In a blog of John O'Sullivan are reported discussions between IR-thermometer manufactures and Alan Siddons and the experiments of Nasif Nahle. [**5**]

We conclude that back-radiation has not been confirmed experimentally. Diagrams in climate literature where back-radiation is given explicitly in W/m^2, such as the so-called K&T diagrams, give the wrong energy balances. The interpretation of the back-radiation fluxes in those papers is missing, the reader has to guess.

Indeed Joseph Postma is wondering, why use the term back-radiation in models when it does not represent a physical phenomenon? [**6**] This paper shows that it is easy to implement multilayer models with the correct interpretation of the SB law.

In a first version only emission by IR-sensitive gases was taken into account, in the present version the emission by all molecules in the atmosphere can be included in the models, as is suggested by Alan Siddons and Joseph Postma: The result is a flexible tool. [**7**] Examples of analyses of the atmosphere are given and enabled to identify errors in K&T diagrams and the like, claiming to represent the global and annual mean heat budget.

One-layer models of the atmosphere.

Hans Schreuder made a compilation of some 60 one-layer models of the atmosphere which can be found in various blogs, papers, textbooks etc.[**8**]
They all use the unfortunate non-physical back-radiation formulation, based on the two-stream model for heat flow. We give here the formulation with the one-stream model with the correct interpretation of the SB law. We show that in the two-stream formulation with the non-physical back-radiation spurious absorption occurs in a symptomatic way.

We repeat here the correct interpretation of the SB law
The underlying philosophy is that pairs of surfaces are identified: a surface **i** with temperature $\mathbf{T_i}$ and a surface **j** with temperature $\mathbf{T_j}$, and $\mathbf{T_i > T_j}$.
The warmer surface **i** emits heat to the colder surface **j.**
Eventually, the absorbed heat in surface **j** is going to be re-emitted to other

surfaces **k** with $T_k < T_j$ and/or to outer space with temperature **0** K, etc.

For $T_i > T_j$: $q(i \rightarrow j) = \sigma(T_i^4 - T_j^4)$ and $q(j \rightarrow i) = 0$ **(1)**

It is to be noted that equation (**1**) describes 2 physical phenomena:
(a) emission of a quantity of heat: $\sigma(T_i^4 - T_j^4)$ by surface **i**
(b) absorption of same quantity of heat: $\sigma(T_i^4 - T_j^4)$ by surface **j**

To prepare the reader for the multilayer model of the atmosphere we start with the one-layer model for both the one-stream and the two-stream implementations. In such models, the atmosphere consists of a single semi-transparent layer, which is modeled by fully transparent holes with normalized cross-section **"1-f "** and the remaining part with normalized cross-section **"f"** is fully opaque.

Fig 1a One-slab model of semi-transparent atmosphere
Implementation A, according to the one-stream heat flow.

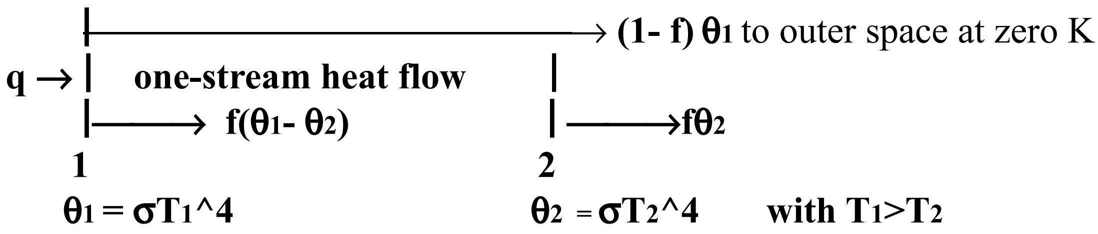
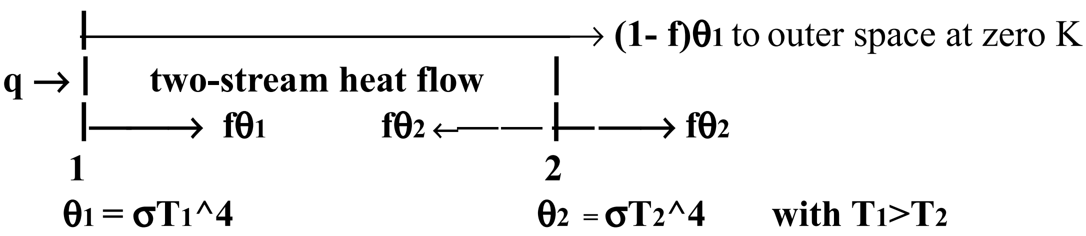


Fig 1b One-slab model of semi-transparent atmosphere.
Implementation B, according to the two-stream heat flow.



In figures **1a** and **1b** the slab is represented by the vertical lines **2.** Lines **1** are the surface of the planet which absorbs in both models a heat flux **q.**
We introduce the variables: $\theta_1 = \sigma T_1^4$ and $\theta_2 = \sigma T_2^4$.
This facilitates the editing as well as the reading of the paper.
Both the flux **q** and the variables **θ** have dimension W/m^2.
We assume steady state conditions.

Note the two basically different implementations:
Implementation **A** : one-stream heat flow $\mathbf{f(\theta_1 - \theta_2)}$ from warm to cold according to the SB equation (**1**).
Implementation **B** : two-stream heat flow, with Prevost-Fourier type of source terms $\mathbf{f\theta_1}$ from level **1** and $\mathbf{f\theta_2}$ from both sides of the slab 2, giving rise to what is called "back-radiation".
It will turn out that this formulation should be avoided, because of spurious absorption in the slab.

In both cases we find for the radiation flux to outer space at zero K:

$$q = (1 - f)\theta_1 + f\theta_2$$

Implementation **A:** $f(\theta_1 - \theta_2) = f\theta_2$

Implementation **B:** $f\theta_1 = 2f\theta_2$

Both **A** and **B** give: $\theta_1 = 2\theta_2 = q/(1-f/2) = 2q/(2-f)$

We could conclude that the two implementations are equivalent.
However, there is an important difference when we observe the heat which is absorbed by the slab and re-emitted immediately.
For model **A** we see from figure **1a**:

$$q_{abs} = f(\theta_1 - \theta_2) = f\,\theta_2 = f\,q/(2-f) \qquad \textbf{(2)}$$

For model **B** we see from figure **1b**:

Emitted by level **2**: $2f\,\theta_2 = 2f\,q/(2-f)$

Absorbed by level **2:** $f\,\theta_1 = 2f\,\theta_2 = 2f\,q/(2-f) \qquad \textbf{(3)}$

Emission and absorption are equal, also in model B because we assume steady state conditions.
It is observed however, that the absorption and the emission are for model **B** twice as big as compared to the model **A!**
*And model **A** is the correct one**,** sufficient to look for the value for **f=1:** the completely opaque slab absorbs and emits **q** and not **2q**!*
Not only thermodynamics but also the algebra does not allow that heat flows from the colder temperature $\mathbf{T_2}$ to the warmer temperature $\mathbf{T_1}$!

Multilayer model consisting of fully opaque layers

It turns out that model **B,** the two-stream implementation with Prevost-Fourier type of source terms giving rise to what is called back-radiation, shows the discrepancy of a too high absorption in a symptomatic way!
This anomaly will be shown for a stack of completely opaque slabs, as a preparation to the multilayer model for the semi-transparent atmosphere.

We consider a stack of **N-1** completely opaque slabs and introduce the variables $\theta_i = \sigma T_i^4$. The ground level **1** has an input heat flux **q**, the level **N** emits in steady state the same flux **q** to outer space at zero K.
Note in figure **2a** the different lines: *fluxes, levels, variables.*

Fig 2a Stack of N-1 completely opaque slabs.
Implementation A with the correct SB law (1), according to the Second Law of Thermodynamics from warm to cold *i.e.* one-stream heat flow.

fluxes:	q $\theta_1-\theta_2$	$\theta_2-\theta_3$	$\theta_3-\theta_4$		$\theta_{N-1}-\theta_N$	θ_N
	→|→	|→	|→	..	|→	|→
levels:	**1**	**2**	**3**		**N-1**	**N**
variables:	θ_1	θ_2	θ_3		θ_{N-1}	θ_N

In steady state conditions we conclude immediately: $\theta_N = q$
For layer **N** from **in=out**: $\theta_{N-1} - \theta_N = \theta_N \rightarrow \theta_{N-1} = 2\,\theta_N \rightarrow \theta_{N-1} = 2q$
In general for level **i** : $\theta_i = (N-i+1)q$
For the ground level **1**: $\theta_1 = Nq$
We see that between all plates the flux is equal to: $\theta_{i-1} - \theta_i = q$
Each plate absorbs and re-emits immediately the same flux: **q**

These results are as could be expected: temperatures towards the ground level increase, and the fluxes remain within the limits of the boundary values.
Each plate absorbs and re-emits a heat flux **q**.

We now analyze the same stack of **N-1** completely opaque slabs by means of model **B** with the two-stream heat flow, Prevost-Fourier type of source terms θ_i in the **two** directions of level **i >1**, as shown in figure **2b**.
We will show that the errors in model **B** are symptomatic: two-stream models with back-radiation should be avoided.

The author is aware of this strong statement: IPCC authors use the two-stream model for radiation analysis.
We will show that it gives spurious numbers for the absorption.
The greater part of universities teach the wrong two-stream model for heat flow. It is a crime against the Second Law of Thermodynamics.

Fig 2b Stack of N-1 completely opaque slabs.
Implementation B with the two-stream heat flow and Prevost-Fourier type of source terms.

fluxes:	q θ_1	θ_2 θ_2	θ_3 θ_3		θ_{N-1} θ_{N-1}	θ_N θ_N
	→|→	←|→	←|→	...	←|→	←|→
levels:	1	2	3		N-1	N
variables:	θ_1	θ_2	θ_3		θ_{N-1}	θ_N

In steady state conditions we conclude immediately: $\theta_N = q$
For layer **N** from **in=out**: $\theta_{N-1} = 2\theta_N \rightarrow \theta_{N-1} = 2q$
In general for level **i** : $\theta_i = (N-i+1)q$
For the ground level **1**: $\theta_1 = Nq$

We see for two-stream implementation **B** the same temperature distribution as for the one-stream implementation **A**.
But the fluxes are different! As well as the absorption in the slabs!
As concerns fluxes, back-radiation defenders say that the net heat flux between the layers in the two-stream model is also : $\theta_{i-1} - \theta_i = q$
They subtract in the results of the two-stream model the two fluxes.
Why not starting with the one-stream model in the first place?

But the **absorption** in the various slabs according to the two-stream model **B** is completely different from the value **q,** which we have found for all slabs in the stack by means of the one-stream model**.**

Consider for example slab **2** in figure **2b**:
Slab **2** emits according to figure **2b**: $2\theta_2 = 2(N-1)q$
Slab **2** absorbs from layer **1** and **3**: $\theta_1 + \theta_3 = Nq+(N-2)q = 2(N-1)q$
In general: slab **(1 < i <N)** emits and absorbs: $2\theta_i = 2(N-i+1)q$

The balances are satisfied, but it is a non-physical result of the two-stream model **B**, that the lower levels absorb these huge amount of heat -and emit it immediately- instead of a value **q** as found in the one-stream model **A.**
For a stack of 20 completely opaque slabs the absorption in slab **2** would be 40 times the absorption in the slab of model **A**!

We conclude that the two-stream heat flow model with Prevost-Fourier type of source terms, gives spurious absorption and should be avoided.

The theory of IPCC authors is wrong, a crime against the Second Law.

Multilayer model of the semi-transparent atmosphere

For a stack of more than 2 semi-transparent layers, the analysis is tedious to carry out analytically. We will derive in this section the simultaneous equations for a multilayer semi-transparent stack according to the one-stream model and solve them using MATLAB.

We model the semi-transparent atmosphere as a stack of grids consisting of fully transparent holes and fully absorbing wires with a normalized cross-section **"1-f "** and **"f"** respectively. The wires of a grid in the stack exchange heat by radiation with the wires of other grids in the stack according to equation (**1**) *i.e.* from the warmer ones to the colder ones.

In transverse direction *i.e.* parallel to the surface of the planet, the dimensions are supposed to be infinite and by symmetry only heat flow by radiation in perpendicular direction has to be considered.
In this way we avoid any discussion about the applicability of SB for exchange of heat by radiation in gases.

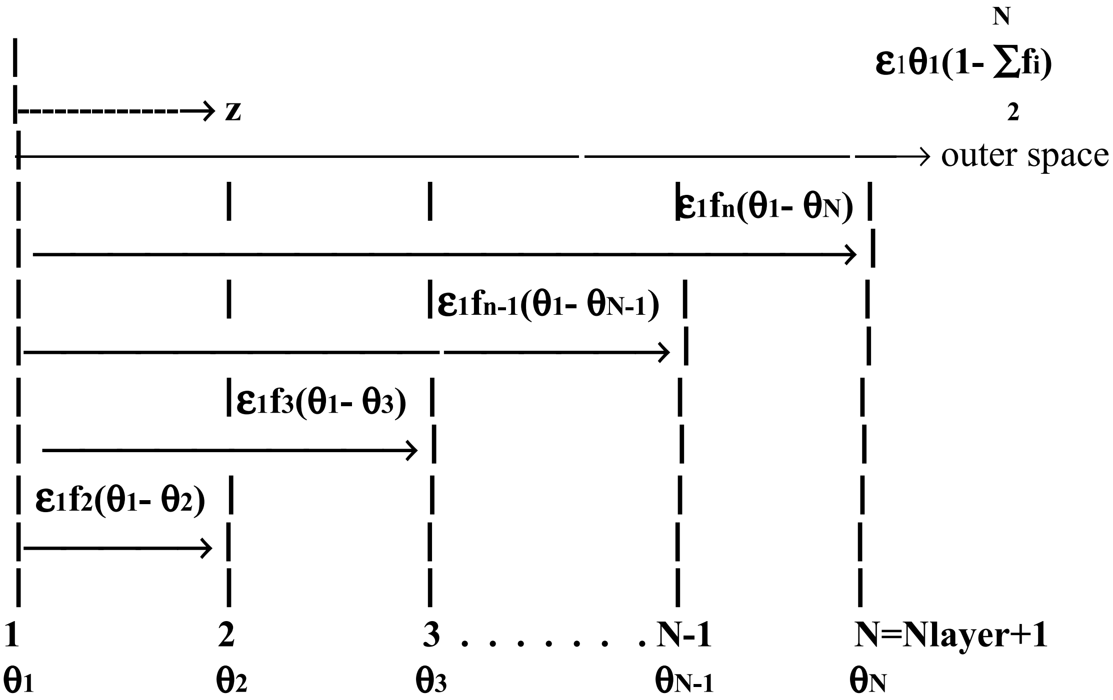
The discussion whether the model is representative for an atmosphere with traces of IR-sensitive molecules can then be postponed until numerical results are produced and judged on their coherency.
The cross section "**f**" of the wire *i.e.* the diameter of the wire multiplied by its length per unit of area of the gauze, can vary for the grids in vertical direction and can be interpreted as an absorption coefficient.
It will be called as such from now on.

In figure **3** is depicted the model with a number of layers, **Nlayer,** and a number of levels, **N=Nlayer + 1**, represented by vertical lines.
For each level **i** we define the variable $\theta_i = \sigma T_i$^4.
Level **1** on the left hand side represents the surface of the planet, level **N** at the right hand side is the tropopause, the upper boundary of the model also called top of atmosphere, **TOA**.

We introduce for each level an absorption coefficient f_i which is a geometrical cross-section, and an emission ε_i which is for a "black" wire equal to f_i. For the ground level f_1**=1** and ε_1 is taken as 1. Implicitly it is assumed that the temperature decreases with height.
That makes the logic in the program more straightforward.
It is noted that for 20 layers there are about 20x20/2=200 pairs to be checked to find out which of the two levels is warmer than the other, in order to

decide in which way the heat flux goes. When a negative dT/dz is assumed over the complete height of the stack, such checks can be omitted.
It would be easy to deal with *e.g.* inversions by introducing a test to define the direction of the heat flux on the basis of the temperature difference, conform the Second Law of Thermodynamics.

Fig 3 Heat fluxes from the surface level 1 towards the (N-1) colder layers and towards outer space at 0 K.



With the parameters $\mathbf{f_i}$ and $\boldsymbol{\varepsilon_i}$ we can apply the SB equation **(1)** between the various levels and to outer space. For the time being, the emission coefficients $\boldsymbol{\varepsilon_i}$ refer to emitting layers in upward direction and the absorption coefficients $\mathbf{f_i}$ to receiving absorbing layers. Since we consider the wire "black" : $\boldsymbol{\varepsilon_i} = \mathbf{f_i}$.
We will also consider short wave (SW) downward radiation from the sun, with the signature of the temperature of the sun and penetrating the atmosphere. This SW radiation in the direction of the earth will be absorbed by a coefficient $\mathbf{a_i}$ depending on the concentration of aerosols, water droplets, ice crystals etc, but re-emitted as LW radiation towards colder layers upwards. For the time being due to the lack of data, $\mathbf{a_i}$ in downward direction is taken equal to $\mathbf{f_i}$. Anyhow, it has shown to be of minor importance where in the atmosphere the incoming SW radiation is converted to an outgoing long wave IR-radiation. But the software is now available to let aerosols, water droplets and ice crystals etc. absorb short wave sunlight and emit it afterward as long wave IR in upward direction towards colder temperatures, with an emission coefficient taking into account those aerosols, water droplets and ice crystals etc.

From figure **3** we define a relation between the flux $\mathbf{q_1}$**,** from the surface level 1, and the variables $\boldsymbol{\theta_i}$ of the various levels:

$$q_1 = \varepsilon_1\theta_1(1-\sum_{i=2}^{N} f_i) + \varepsilon_1\sum_{i=2}^{N} f_i(\theta_1-\theta_i) = \varepsilon_1\theta_1 - \varepsilon_1\sum_{i=2}^{N} f_i\,\theta_i \qquad (4)$$

This is a first relation between the **N** variables $\boldsymbol{\theta_i}$.
We can use the structure of the expression (**4**) to write in a recurrent way the fluxes emitted from levels **2,3,.. N**.
The flux emitted respectively absorbed by level **2** becomes:

emitted by **2:** $\quad \varepsilon_2\theta_2 - \varepsilon_2\sum_{i=3}^{N} f_i\,\theta_i$

absorbed by **2:** $\quad \varepsilon_1 f_2(\theta_1-\theta_2)$

This gives a second relation between the **N** variables $\boldsymbol{\theta_i}$ **:**

$$\varepsilon_2\theta_2 - \varepsilon_2\sum_{i=3}^{N} f_i\,\theta_i = \varepsilon_1 f_2(\theta_1-\theta_2) \qquad (5)$$

We can repeat this process for levels **j=3,N** giving another **N-2** relations.
In general out from level j respectively into level j:

emitted by **j:** $\quad \varepsilon_j\ \theta_j - \varepsilon_j\sum_{i=j+1}^{N} f_i\,\theta_i \qquad$ **(6)**

absorbed by **j>1:** $\quad f_j\sum_{i=1}^{j} \varepsilon_i\,(\theta_i-\theta_j) \qquad$ **(6a)**

We can assemble (**6**) and (**6a**) into an upper-triangular matrix **OUT** respectively into a lower-triangular matrix **INTO,** both of order **N**x**N.**
As an example we give the matrices for 3 layers *i.e.* 4 levels:

$$\mathbf{OUT}*\theta = \begin{vmatrix} \varepsilon_1 & -\varepsilon_1 f_2 & -\varepsilon_1 f_3 & -\varepsilon_1 f_4 \\ 0 & \varepsilon_2 & -\varepsilon_2 f_3 & -\varepsilon_2 f_4 \\ 0 & 0 & \varepsilon_3 & -\varepsilon_3 f_4 \\ 0 & 0 & 0 & \varepsilon_4 \end{vmatrix} \begin{vmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{vmatrix}$$

$$\mathbf{INTO}*\theta = \begin{vmatrix} 0 & 0 & 0 & 0 \\ \varepsilon_1 f_2 & -\varepsilon_1 f_2 & 0 & 0 \\ \varepsilon_1 f_3 & \varepsilon_2 f_3 & -(\varepsilon_1+\varepsilon_2)f_3 & 0 \\ \varepsilon_1 f_4 & \varepsilon_2 f_4 & \varepsilon_3 f_4 & -(\varepsilon_1+\varepsilon_2+\varepsilon_3)f_4 \end{vmatrix} \begin{vmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{vmatrix}$$

The **N**x**N** triangular matrices can be generated automatically for given values of f_i and ε_i for **i = 1,N.**
We build the system equation by comparing at each level
flux out and **flux in:**

K = OUT-INTO → K*θ = RHS **(7)**

K **:** square matrix of order **NxN.**
θ **:** vector of unknowns of order **N**
RHS : right hand side vector of thermal fluxes into the system, of order **N**

We can construct **RHS** from the incoming flux from outer space, at station **N,** also called top of atmosphere**: TOA.** This radiation has the signature of the temperature of the sun and travels downwards from warm to cold.
If we call this incoming flux **qtoa**, and assume the absorption coefficients a_i for **i=2,N** for the downward incoming radiation for each station:

$qtoa_N=qtoa$
for i=N:2
$RHS_i=a_i*qtoa_i$ **(8)**
$qtoa_{i-1} = qtoa_i - RHS_i$
end

And at level **1**, the ground level we find: :
$RHS_1 = qtoa_1$ $(q_1 = RHS_1)$ **(9)**

For the time being, because of the lack of data, the coefficients a_i are taken equal to the coefficients f_i. When analyzing K&T diagrams we take the total SW absorption in the atmosphere and distribute it according to the distribution of the long wave absorption coefficients at the various levels.

The boundary conditions are already incorporated, since the outer space temperature in the SB law is assumed to be at zero K**.**
The solution of **(7)** can be written as:

$$\theta = inv(K)*RHS \quad \textbf{(10)}$$

In appendix **1** the **K** matrix for one slab (2 layers) is given and solved

analytically, giving the classical results.
The matrix **K** is a square symmetric matrix and for all applications *i.e.* number of layers from 1 up to 100, the matrix could be inverted.
It is intended to incorporate in this no back-radiation program, the line by line method to define more accurately the absorption and the emission as function of atmospheric parameters and to make a distinction between the absorption of downward short wave absorption by aerosols, water droplets, ice crystals etc, and the upwards long wave absorption by IR-sensitive molecules.
We will see that the model of the atmosphere by means of a stack of grids gives coherent results. Diagrams of the K&T type, concerning the global and annual mean value for **qtoa** and the global and annual surface temperature refer to different situations, are not of big use. They will be analyzed merely to show the fact that the use of the two-stream model and thereby back-radiation gives contradictory results because in that implementation the atmosphere absorbs and re-emits a factor of about 3 too high.

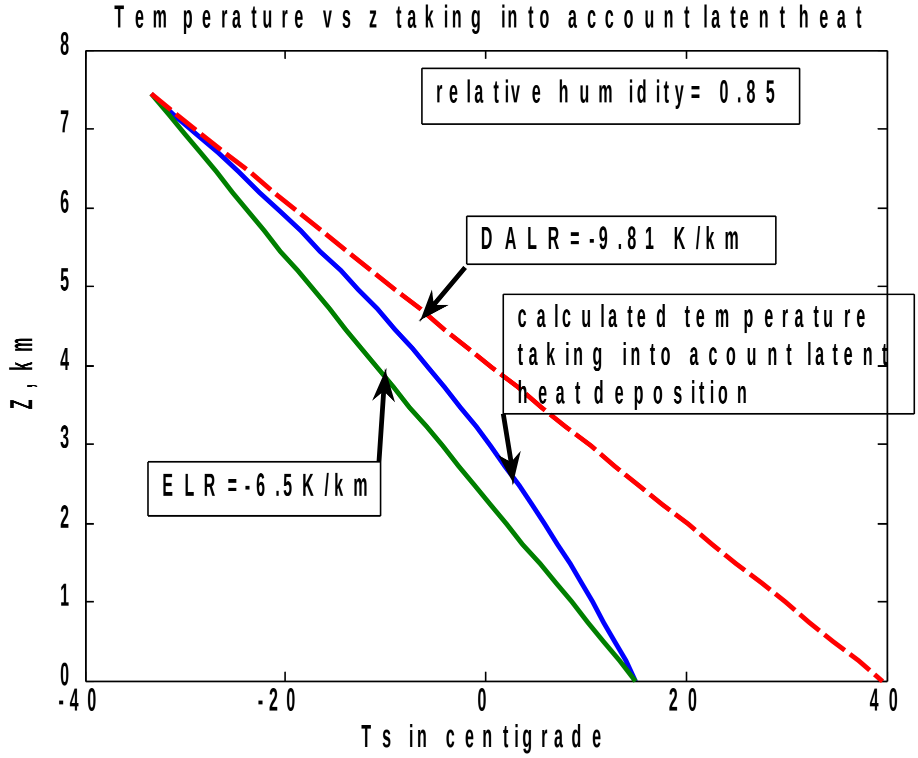
Potential of the radiation mechanism of the atmosphere.

It occurs that applying SB on pairs of surfaces according to the one-stream model, balances of net heat fluxes can be established at each level, giving rise to a multilayer model with no violation of the Second Law of Thermodynamics. Back-radiation does not show up.
The IR-sensitive gases put, however, a semi-transparent screen over the earth in a way that the outgoing long wave radiation is hindered. In order that the same heat flux is send to outer space, the temperature increases!
It is not correct to call that increase of the surface temperature - towards more convenient and comfortable temperatures - the "green house effect". And this for two reasons.

(1) In the first place the temperature in a nursery green house is not that much higher because outward IR-radiation is hindered, but mainly because the heat loss due to convection is hindered. Experimental evidence about that phenomenon goes back to the beginning of the 1900's, with the experiments of Wood, and more recently these experiments have been repeated by many others all over the world.

(2) The fact that the temperature decreases for higher altitudes is mainly due to the adiabatic lapse rates, the dry one, **DALR = -g/**c_p **= -9.81** K/km and the saturated one which is, in absolute value, lower because of the fact that latent heat of condensation is released when saturated air is rising, **SALR= -5** K/km**.** From figure **4** we see that for **z=0** and the given humidity which is nearly saturated, **dT/dz** is close to **SALR** and for higher heights when all vapor is condensed **dT/dz** is approaching to **DALR**.

Figure 4 Calculated temperature distribution according to appendix 3.(American Meteorological Society)



We will assume the environmental lapse rate: **ELR = - 6.5** K/km**,** to define a temperature distribution in the atmosphere.
However, to show the influence of the temperature distribution we identify the calculated solid blue line, defined by correlations from the American Meteorological Society, with the logo: **AMS**. (see Appendix 3)

We will use the model as presented in this paper to show that the straight forward formulation of the SB law gives rise to a matrix relation between incoming radiation at the **TOA** and the outgoing **OLR**, which results into a flexible tool to show the ample margins of the heat housekeeping of the planet. We repeat the main equation **(7)** of the model:

$$K*\theta = RHS \qquad (11)$$

K is a **NxN** matrix, based on absorption and emission distribution, relating the unknowns $\theta_i = \sigma T_i^4$, for **i=1,N** to the **RHS** defined by **qtoa** as given by **(8)** for the atmospheric terms and **(9)** for the surface term, assuming steady state conditions.
We will use equation **(11)** to determine the necessary **RHS** to obtain a known measured temperature distribution**,** defined by the surface temperature **Ts** and the environmental lapse rate **ELR**:

$$T_{LR}(z) = T_s + ELR*z \;\rightarrow\; T_{LRi} = T_s + ELR*z_i \qquad (12)$$

In figure **4** is given the calculated real temperature distribution taking into account the release of latent heat, but we will use in the analysis: **ELR.**
The vector $\mathbf{T_{LR}}$ can be converted, component per component, into a vector of variables $\boldsymbol{\theta_{LR}}$ with dimension W/m^2.

$$\theta_{LRi} = \sigma T_{LRi}^4 \qquad \textbf{(13)}$$

By applying **(9)** in a reversed way, see also Appendix **2,** we find a vector of heat fluxes $\mathbf{q_{LR}}$ related to $\boldsymbol{\theta_{LR}}$:

$$q_{LR} = K * \theta_{LR} \qquad \textbf{(14)}$$

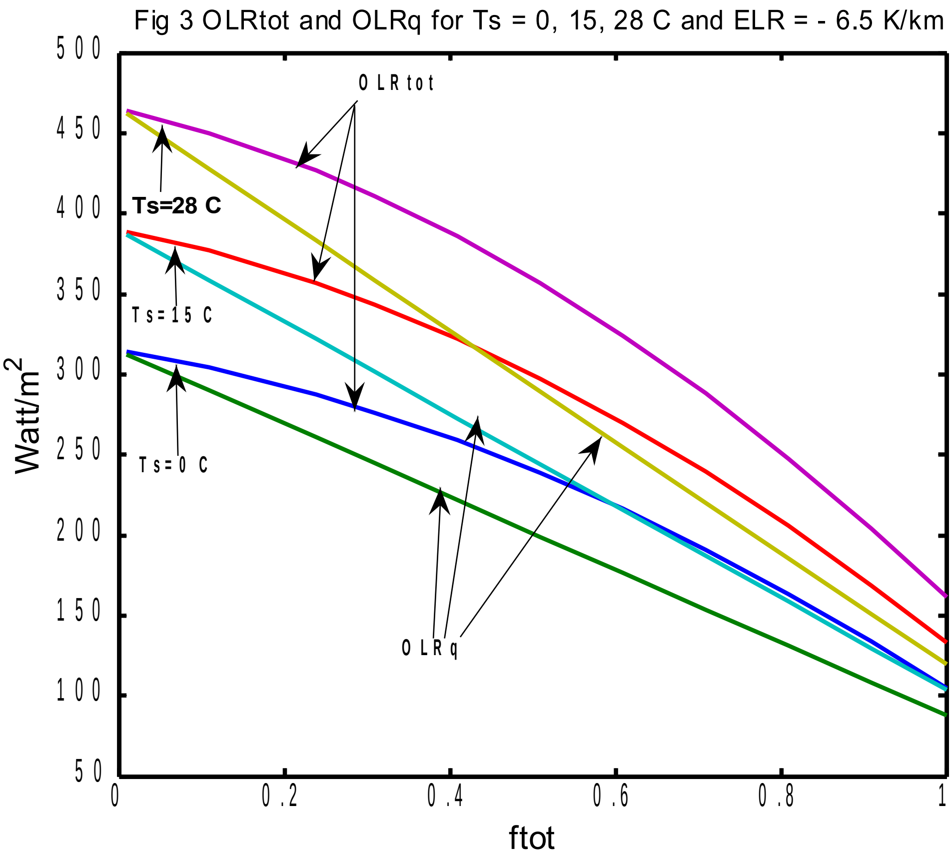
What represent the **N** components of the vector $\mathbf{q_{LR}}$?

They are the mismatches at each level of the heat balance according to the radiation mechanism, for the existing temperature distribution $\mathbf{T_{LR}}$ defined by surface temperature **Ts** and environmental lapse rate **ELR**.

In other words, for the assumed temperature distribution, they are the possible fluxes of heat which can be evacuated to outer space by radiation! Not only the most important surface term, $\mathbf{q_{LR1}}$, but also terms along the complete height of the model $\mathbf{q_{LRi}}$**, i=2,N** due to all kind of mechanisms: absorption of incoming sun light, deposit of sensible and latent heat by a convection mechanism, absorption and re-emission by solid particles due to volcanoes, etc. And not only transport of heat terms in vertical direction due to convection of sensible and latent heat, but also the effect of transverse transport by means of wind!

In still other words, the first component of the vector $\mathbf{q_{LR}}$ is the maximum possible outgoing IR surface flux, by the radiation mechanism.
And the sum of all components of the vector $\mathbf{q_{LR}}$ is the maximum possible, outgoing IR surface flux combined with other heat sources in the atmosphere, by the radiation mechanism. We will see that the measured temperature distribution *i.e.* **ELR** and **Ts** give ample margins to the earth to evacuate the heat receiving from the sun.

Fig5



OLRq (only due to a surface flux) $= q_{LR1}$ **(15)**

OLRtot (due to surface flux and all other atmospheric heat deposits) $= sum(q_{LR})$ **(16)**

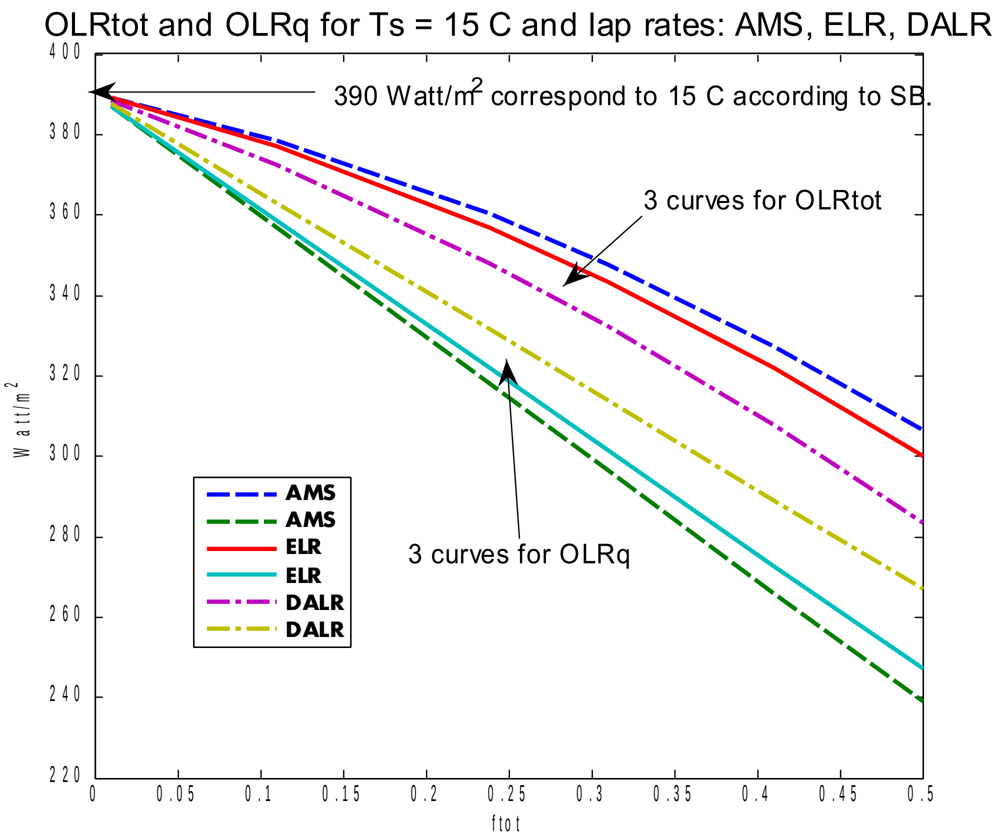
In figure **5** the results are given for a model with 20 layers for **H = 10 km, ELR = -6.5** K/km**,** and for 3 surface temperatures **Ts**=**0, 15** and **28** C, as function of a linear distribution of the absorption coefficients in the atmosphere with **ftot**= $\sum f_i$.

We have used also more layers, but the results of 20 layers are more or less converged.
We have tested the effect of the distribution of the absorption coefficient. The effect was small, we continue to take a linear distribution.
We have also varied the distribution of the thickness of the layers and concluded that the distribution of the thickness of the layers is small as could be expected. For the purely radiation problem, the distribution of the thickness of the layers can be arbitrary. When we want to satisfy the temperature distribution a variation of the thickness according to a geometrical series with ratio 1.3 gives stationary results, but the influence is small.

We see that the influence of the surface flux is dominant, but of course a surface flux alone cannot give an exact linear temperature distribution according to the lapse rate, the contribution of the atmosphere acts as a trimmer to arrive at the exact linear distribution. It is indeed the fourth power in SB and the emission coefficient of the ground level as compared to the atmosphere which makes the temperature of the surface dominant.

Another exercise is given in figure **6**, where for **Ts=15 C** the same results for **OLRtot** and **OLRq** as function of the absorption coefficient **ftot** are given. But we have used different lapse rates: **ELR, DALR** and the **AMS** temperature distribution for the relative humidity **0.85** given in figure **4.**
The dependence of **ftot** on the humidity is not included**!**
For **ftot → 0**, the curves for **ORLq** and **OLRtot** intersect at a level
ORLtot=ORLq= σ(Ts+273)^4 = 390 W/m^2 for **Ts = 15** C**.**
We see the influence of the temperature distribution on **ORLq** and on **OLRtot**. The difference is small between **AMS** and **ELR**: we will continue to use **ELR**, also because **ASM** depends on the humidity.
These figures **5** and **6** show that there is an ample margin of the planet earth to evacuate the heat she receives from the sun back to outer space,
at least for the mediate surface temperatures due to a mediate **qtoa**.

Figure 6

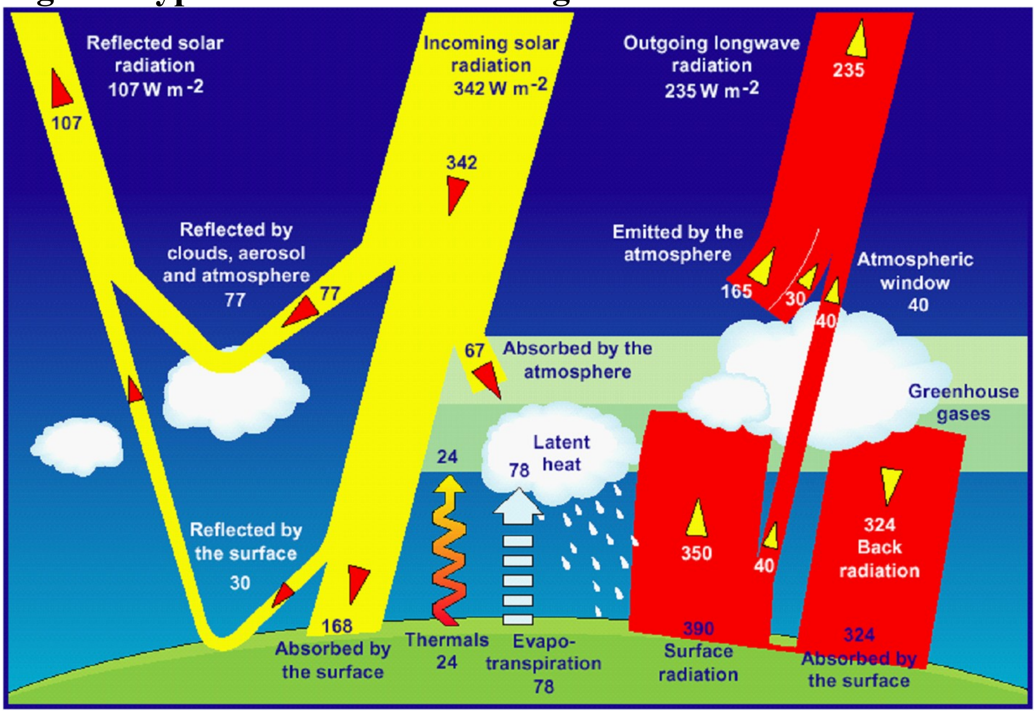


For tropical zones with sea surface temperatures around **28** C and a high absorption due to a high relative humidity the margin is less.
We come back on that issue in appendix **4** on Hot Towers.

Global and annual mean balances of heat.

In the frame of IPCC several authors have made global and annual mean energy balances, which are known as K&T diagrams. They are all based on the two-stream model with back-radiation which according to our analysis should be avoided. It is a crime against the Second Law of Thermodynamics. Besides the numbers given to that back-radiation are wrong, since in the two-stream formulation spurious absorption occurs.
It is assumed in those IPPC papers that the global and mean flux at **TOA**, including albedo is about **235** W/m2 and the global and annual mean temperature is **15** C. It is to be noted immediately that the two events are not connected physically, as is already argued by Joseph Postma, and the balances are of little use. [**6**]

Fig 7 A typical so-called K&T diagram



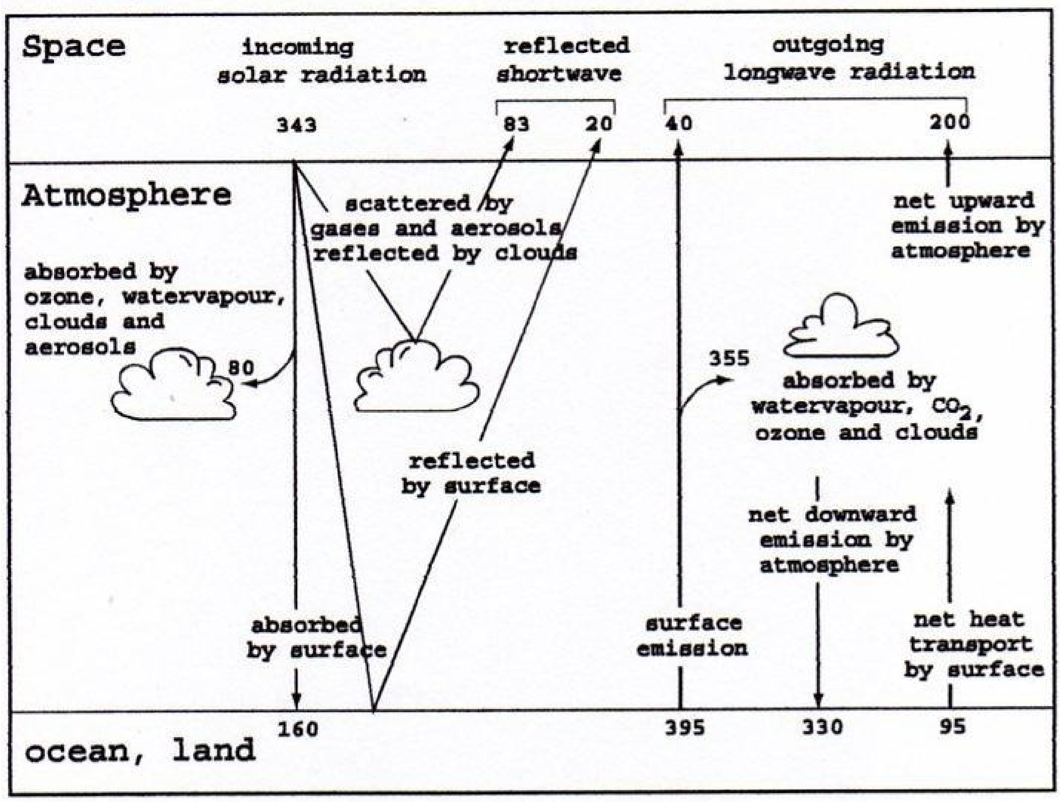
We observe, in the right hand side lower corner, two fat arrows.
One upwards, called surface radiation, of **390** W/m^2, and the other one downwards, called back-radiation, of **324** W/m^2**.**
The upward **390** W/m^2**,** would be the surface radiation with a sky free from IR-sensitive molecules**.** Indeed it is the intersection in figure **6** of the curves **OLRq** and **OLRtot** with the vertical axis **ftot=0**!
Under these conditions at **Ts=15** C=**288** K and a clear sky without any IR-

sensitive molecules the earth would emit **390** W/m^2 to outer space at **0** K**,** according to SB**.** But the earth is only receiving **235** W/m^2 at **TOA,** as the yearly average over the globe**!**
So we have more or less identified one of the big arrows in K&T**:** it is an imaginary value, it only happens for **ftot=0** and **qtoa =390**
For the other fat arrow of **324** W/m^2, called back-radiation, which does not exists since it would be a crime against the Second Law of Thermodynamics, things are more difficult to explain**!**

In another K&T type of diagram, figure **7a,** by Rob van Dorland of KNMI, Netherlands, with slightly different numbers, it is even said and shown more explicitly: the surface is emitting **395** W/m^2 of which **355** are absorbed by the atmosphere which is sending back **330** as back-radiation to warm up the surface!

Figure 7a

From "Radiation and Climate. From Radiative Transfer Modeling to Global Temperature Response" by Rob van Dorland, doctoral thesis November 1999, University of Utrecht, Netherlands. Fig 2.3; Global and annual mean energy balance of the climate system in W/m^2



Still another global balance is given by Ferenc Miskolczi. [9]

Figure **7b**

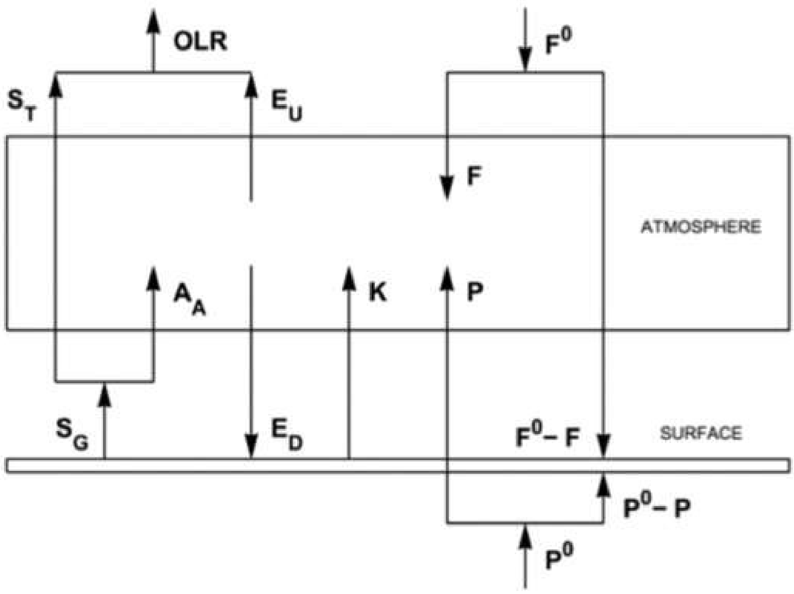


Table 2. Ranges and the global averages of the different physical quantities.

Fluxes are in Wm-2, u is in prcm, tA is in K.

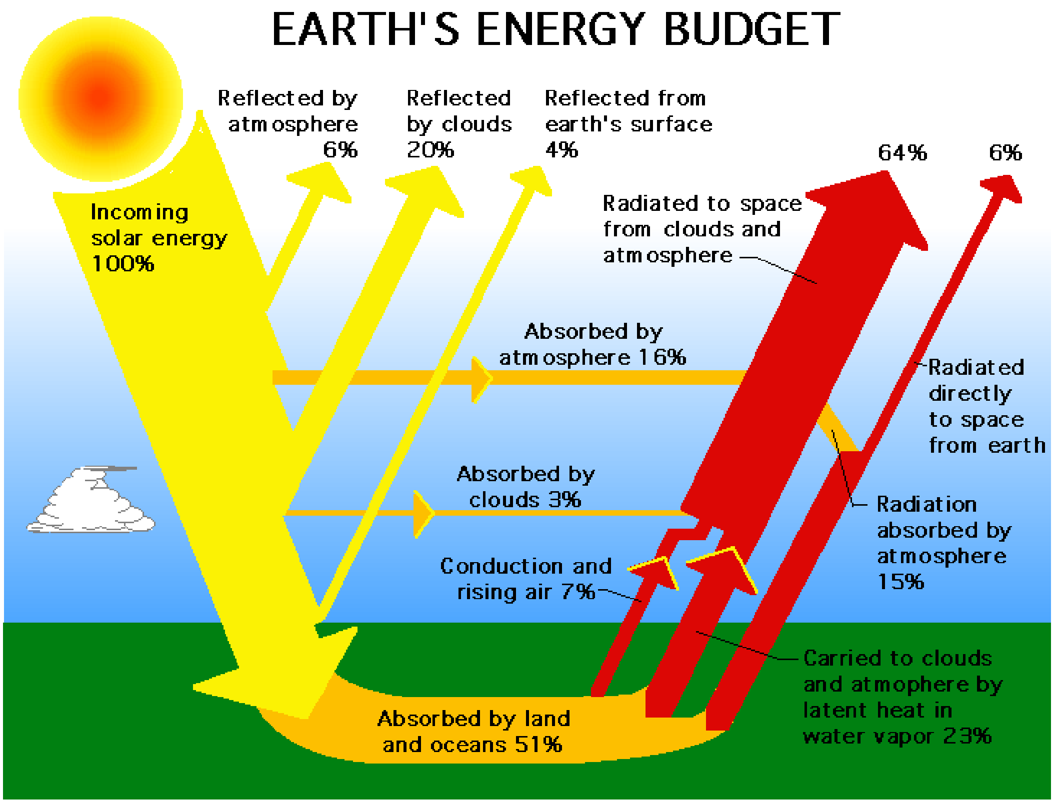
Quantity	*Minimum*	*Maximum*	*Global Average*	*GAT*
tA	*232.25*	*309.62*	*285.34*	*286.04*
SU	*164.98*	*521.10*	*381.88*	*379.64*
u	*0.0507*	*6.836*	*2.533*	*2.637*
EU	*83.74*	*256.71*	*188.94*	*192.7*
ED	*103.35*	*429.69*	*308.70*	*310.49*
TA	*0.0497150*	*0.391204*	*0.173344*	*0.15422*
ST = SUTA	*22.246*	*111.92*	*61.094*	*58.54*
OLR = EU + ST	*150.64*	*297.62*	*250.05*	*251.25*
$\tau = -\ln(TA)$	*0.9385*	*3.0014*	*1.8736*	*1.8693*

In the figure and the table back-radiation is indicated by **ED** and the long wave absorption by the atmosphere by **AA**.
And **AA** ≅ **ED** according to Ferenc Miskolczi.

Arguments are forwarded not to interpret the K&T imaginary value of **390** as what would happen in reality - which is very true as we have demonstrated for the one-layer model! - but one has to take into consideration the value **324** of non existing back-radiation!

NASA has ceased to mention the crime against the Second Law of Thermodynamics of back-radiation by IPCC and has now an own K&T type of diagram for the global and annual mean energy budget: figure 5c. NASA has put the energy budget in percentages. Taking 100% equal to **342** W/m^2, we obtain more or less the same numbers as the other IPCC authors. The heat flux by radiation is **(15+6)=21%** giving **72** W/m^2.

Figure 7c



If we continue with the K&T values, should we -like NASA- consider the difference **390-324=66** as the net surface flux? [**10**]
When we look in more detail, the anomalies become evident.

We take from figure **7** the following K&T data:

net surface heat flow	**qsurfKT**	=	**66**	**(=390 - 324)**
atmospheric SW absorption	**qatmSWKT**	=	**67**	
thermals to atmosphere	**qthermalKT+**			
evaporation to atmosphere,	**evapoKT=**			
vertical and transverse	**qconvKT**	=	**102**	
TOTAL	**qtoaKT**	=	**235**	

From fig 7a we have the data of Rob van Dorland, we call them VD data:

net surface heat flow	**qsurfVD**	=	**65**	**(=395 - 330)**
atmospheric SW absorption	**qatmSWVD**	=	**80**	
thermals to atmosphere	**qthermalVD+**			
evaporation to atmosphere	**evapoVD=**			
vertical and horizontal	**qconvVD**	=	**95**	
TOTAL	**qtoaVD**	=	**240**	

From fig 7b we get data of Ferenc Miskolczi**,** we call them FM data.

net surface heat flow	**qsurfFM**	=	**63**	**(=378 - 315)**
atmospheric SW absorption	**qatmSWFM+**			
thermals to atmosphere	**qthermalFM+**			
evaporation to atmosphere,	**evapoFM=**			
vertical and transverse	**F+K+P**	=	**189**	
TOTAL	**F° = qtoaFM**	=	**252**	

(S_G=378, E_D= A_A =315, F+K+P= 189, S_T/S_G=0.167, see figure **7b)**

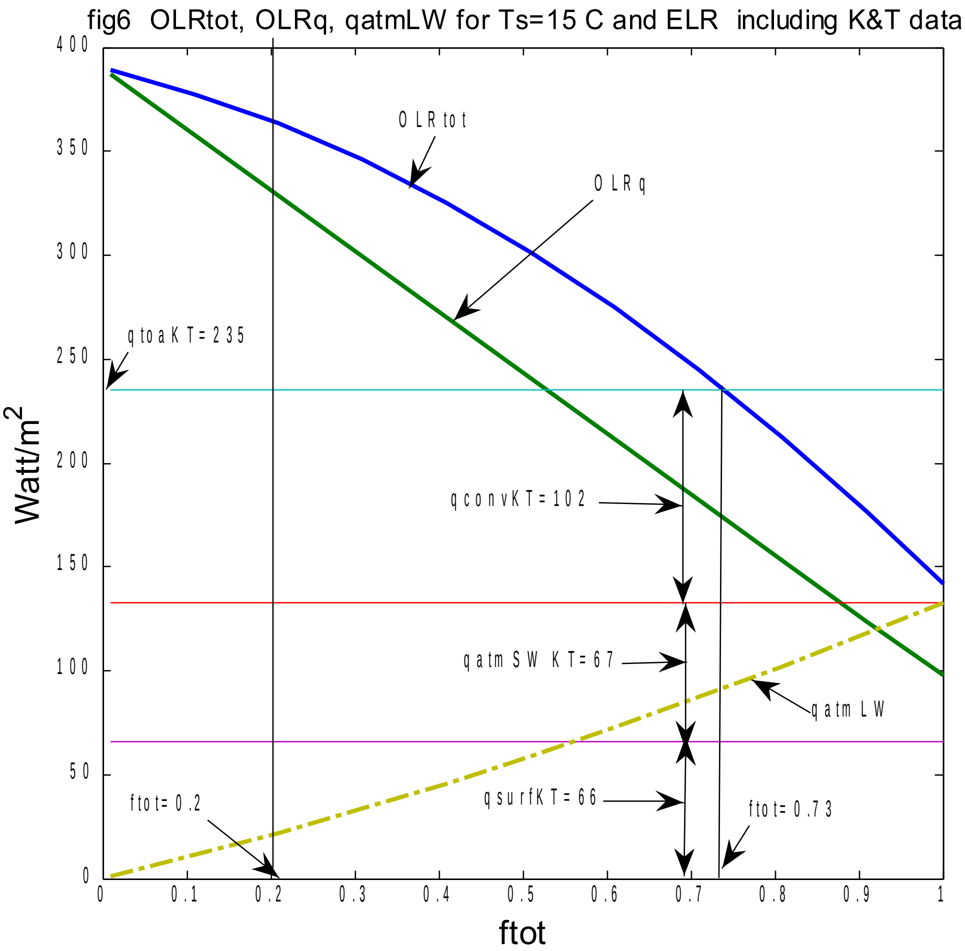
The three sets look similar. There are some slight variations, but in all three we see the huge number for back-radiation, **324**, **330** and **315** for K&T, VD and FM respectively.

Let us take 20 layers in a picture like in fig **5** and **6**, for a temperature distribution **TS = 15** C and **ELM = - 6.5** K/km**,** and we add in figure **8** horizontal lines for the K&T values from the balance :**qsurfKT = 66, qatmSWKT = 67, qconvKT=102** with **qtoaKT=qsurfKT+qatmSWKT+qconvKT = 235.**

The intersection of the horizontal **qtoaKT=235** with **OLRtot** indicates that K&T have used an absorption which in our model would be **ftot=0.73**.

Included is a line with at **ftot=0.2** as proposed by Nasif Nahle. [**11**]

Figure 8



Now let us look to the value of **qtoaKT**. It consist of three terms, and the authors of figure **7, 7a, 7b** have given numbers to them such that the balances are correct.

qsurfKT : Follows from the subtraction of two imaginary numbers as indicated in red in the K&T balance.

qatmSWKT: It is claimed that it represents the absorption of SW sunlight by the atmosphere by aerosols, etc. Such particles will also augment the emission!
This number in the VD data is 80 instead of 67, a difference of 13/67=19%.

qconvKT: It is claimed that it represents the global and annual heat flux by vertical and transverse convection of sensitive and latent heat. The term should be higher, it should also take into account the contribution of hot towers (see appendix 4**)**.

All three are so called "miscellaneous" items, in vogue by bookkeepers to make a false balance correct.

Let us try to find the non-existing back-radiation in our one-stream model. We copy equation (**4**):

$$q_1 = \varepsilon_1\theta_1(1-\sum_{i=2}^{N} f_i) + \varepsilon_1\sum_{i=2}^{N} f_i(\theta_1 - \theta_i) \qquad \textbf{(17)}$$

Or re-arranging algebraically:

$$q_1 = \varepsilon_1\theta_1 - \varepsilon_1\sum_{i=2}^{N} f_i\theta_i \qquad \textbf{(17a)}$$

The two equations **(17)** and **(17a)** are algebraically equivalent!

The version (**17**) is the correct way that SB should be interpreted as shown in figure **3**: one-stream heat flow from warm to cold according to the Second Law of Thermodynamics.
The flux q_1 is the heat flux leaving the ground level **1**.
The version **(17a)** is only an algebraic re-arrangement of equation **(17).**

*And this is the unfortunate error the IPCC authors make, trying to give a physical interpretation to the terms of the algebraic re-arrangement, instead of using SB in a correct way as indicated by equation **(1).***

The first term of equation (**17a),** for $\varepsilon_1 = 1$, is θ_1**:** it would be the surface flux as mentioned in K&T type of diagrams:
for **Ts=15 C,** $\theta_1 = \sigma(273+15)^4 = 390$ W/m^2.

It is an imaginary heat flux which the surface of the planet at a temperature **Ts = 15 C**, would emit to outer space at **0 K**, if and only if, not hindered by IR-sensitive gases. It would be indeed a steady state equilibrium. For **ftot>0** the real flux leaving the surface is lower.
But IPCC authors interpret the **390** as the real flux, let it absorb nearly completely by the atmosphere which is sending it back to earth.
And not even half upwards!

The second term of equation **(17a),** has as emission coefficient ε_1 of the surface, it is not back-radiation since it has not the emissivity of the higher layers! It is the result of algebraic splitting up of equation **(17),** and next trying to give a physical interpretation to the separate terms!
Some might try to sell it to the reader as back-radiation!

We have already a plot of the term in figure **5, 6** and **8**, since equation **(17)** represents **OLRq =q1.** So what is sold to the public as back-radiation is the difference between the horizontal line at a height of what in K&T type of diagrams is called the imaginary flux of **390** as discussed above, and the line **OLRq**. And this value is even far from the claimed value of **324** for a non-existing back-radiation by the IPCC authors.

Another quantity which is used by back-radiation believers, IPCC alarmists like K&T, VD and others as well as skeptics like FM and others, is **qatmLWKT,** the absorption of LW radiation emitted by the earth.
It is different from the SW absorption **qatmSWKT** due to absorption by aerosols etc. of the incoming short wave sunlight.

Not much attention has been given to those figures by the skeptics, they have been more astonished by the huge values of back-radiation.
It might be clear by now for the reader that the two-stream model of heat flow by IPCC authors gives spurious results for the absorption and it has caused the huge numbers for back-radiation.

According to figure **7** from IPCC authors:

qatmLWKT = **back-radiation -qatmSWKT = 324-67= 257.** **(18)**

Our model is transparent and the reader can see from figure **3** and the correct equation (**17**) that **qatmLW,** the amount of heat leaving the surface and absorbed by the atmosphere, is the second term of (**1**7) plus similar terms emitted by level **j=2,N** to be absorbed by layers **k>j.**
The terms are already assembled in the lower triangular matrix **INTO:**

qatmLW = sum **(INTO*θ)** **(19)**

The quantity is given in figure **8** as function of **ftot,** indicated by **qatmLW.**
We see that for our multilayer one-stream model, it is about **85** for the K&T absorption of **ftot=0.73**, a factor **3** smaller than **257,** a number that is based on the two-stream model with the back-radiation, equation **(18).**
For the single slab models the discrepancy was a factor **2** as indicated by equation **(3).**

Ferenc Miskolczi probably did not know what to do with those huge numbers which his (former) colleagues experimentalists at NASA gave him and he started to look for similarities.

In that search he invented first applications of the Kirchhoff principle by saying that absorption by the atmosphere of the long wave radiation (**A_A** in his nomenclature, **qatmLW** in ours) is about equal to the back-radiation (**E_D** = **315** in his nomenclature, no name in ours because it does not exists):
$A_A \cong E_D$, according to Ferenc Miskolczi.
But as has been explained above, the absorption of the long wave radiation is much lower than the value obtained from imaginary numbers of back-radiation provided by the experimentalists and/or computer programs based on the two-stream heat flow as used by IPCC authors
In a more recent paper by Ferenc Miskolczi : Energy &Environment, Vol **21**,No **4**, **2010**, Paradigms in Climate Research, "*The stable stationary value of the earth's global average atmospheric Planck-weighted greenhouse gas optical thickness"* , the Kirchhoff argument for the relation that **$A_A \cong E_D$** has been abandoned. Ferenc Miskolczi now thinks that the statement has already been invented by Prevost in a paper back in 1791. As already mentioned in the introduction back-radiation was the subject of philosophical discussions between Prevost and Fourier, 200 years ago!

In fact Ferenc Miskolczi probably took the relation from equation **(3),** in the one slab model where the absorption in the slab is equal to the imaginary heat flux of the back-radiation believers. It is a factor **2** too high for the two-stream one-slab model. The discrepancy between the two formulations seems to be a factor **3** for a semi-transparent atmosphere.

The conclusion is that the numbers concerning absorption given by the IPCC authors are not correct since they are obtained by a two-stream model for heat flow with a phantom back-radiation, a crime against the Second Law of Thermodynamics. The reason is the spurious absorption in the two-stream formulation.
Those numbers might also have been invented just to try to make the global and annual mean **qtoa=235** to coincide with a global and annual mean surface temperature of **Ts=15** C. The fact that different authors from different continents -but with an IPCC back ground- come up with the same numbers, does not mean that there is a world wide consensus.
Since the numbers are wrong and not representing a real situation, it looks more as a world wide conspiracy.
Indeed, the global and annual mean heat budget diagrams do not represent a real situation, but seem to be used by IPCC to send alarming messages to the general public.

In fact, for the greater part of the surface of the earth, both days and nights are too short to reach twice every 24 hours a steady state situation!
See again Nasif Nahle's New Concise Experiment on Backradiation. [**11**]

Back-radiation does not exists, not during the day and not during the night, as long as the air temperature is higher than the surface temperature.
Inversions exists after a clear night, certainly, but with a height relatively small since it is based on conduction.

Therefore:
- since back-radiation does not exists, which is confirmed by our one-stream model based on the correct application of SB taking into account the Second Law of Thermodynamics, and
- the absorption by the atmosphere, which the IPCC authors claim to have calculated by a two-stream model or to have measured on the basis of back-radiation is wrong by a factor 3,

we might as well conclude that the absorption coefficients of the order of **ftot=0.73** to **ftot= 0.8** are not correct. They have not been mentioned by K&T and by VD, the author found them from the fluxes in figure **7** and figure **7b**.

Ferenc Miskolczi might be correct that the global and annual optical depth is constant because, according to him, more CO2 means less water vapor.
But the value for the optical depth of **1.87** is not correct, it would mean **ftot=1-ln(1.87)= 0.8459.** As concerns his claim that the phantom back-radiation is about equal to the absorption of LW radiation by the atmosphere (**$E_D \cong A_A$**), the relation might be true but numbers are not correct.
He just took the relation from the two-stream one slab model **B** and ended up with a value for **A_A** a factor 3 too big, and equated it to the non existing back-radiation **E_D**, or the other way around, who knows?

A value of **ftot = 0.2** is much more realistic as proposed by Nasif Nahle.
This value of **ftot = 0.2** is for a water vapor concentration of **2.5** % and includes the contribution of **0.0017** for **390** ppm CO2.
Doubling the CO2 to **780** ppm could give **ftot=0.2017.** The temperature on earth will hardly increase, but plants on earth will grow faster to help to feed the growing world population.
The critical reader will say that the vertical line for **ftot=0.2** in figure **8** would indicate a surface flux of nearly **390** W/m^2**,** for the conditions of a global and annual average of only **235**.
But the **390** number follows from an assumed steady state surface temperature of **15 C**. The present model gives the **possible** heat which can be evacuated to outer space by radiation for the surface temperature **Ts=15** C and **ELM = -6.5** K/km.

The mechanism of how that heat arrives at **TOA** can be anything:
absorption of SW by the atmosphere (water droplets, ice crystals, dust) and convection of latent and sensible heat by means of thermals in vertical

direction and wind in horizontal direction.
Such mechanisms are underestimated by the IPPC authors. [**12**]

Besides, the global and annual mean qtoa of **235** as used by the IPCC authors, does not correspond to the global and annual mean surface temperature **Ts=15** C, for the simple reason that the day and night on earth are too short to reach in 24 hours twice steady state conditions, not during the days nor during the nights.
It is indeed a shame that IPCC authors invent back-radiation and a huge absorption coefficient of **ftot** between **0.72** and **0.84** to make a non realistic balance to alarm the public. It is absurd to try to reduce the CO_2 concentration. It is a harmless gas and it is not the reason of climate variations. The CO_2 is needed as food for plants to feed the increasing world population.

Conclusion

The model of this paper, to consider the atmosphere as a stack of grids and a one-stream model for heat has given coherent results. It enabled the author to identify errors of IPCC authors who - with a two-stream model - claim that back-radiation by IR-sensitive gases is heating up the planet.

But back-radiation is a crime against the Second Law of Thermodynamics. Results from two-stream models for heat flow made the IPCC authors to conclude that the absorption coefficient of the atmosphere would be around **ftot=0.8**, while other authors give **f=0.2** for **2.5%** water vapor, including the contribution of **0.0017** from **390** ppm CO2.

It seems that the computer programs used by the IPCC authors based on a model with two-stream heat flow and thereby Prevost-Fourier type of source terms give rise to spurious absorptions and thereby so called back-radiation. The two-stream model should be avoided.
Those computer programs and computer runs need to be revisited.

Experimental results which various authors have used might have been offered to them as being a proof of back-radiation.
Engineers said "it is a crime against the Second Law"!
That should have waken up those authors.
Experimentalists who have furnished the data for back-radiation should reconsider their measurement techniques.

The global and annual mean energy budgets as presented by IPCC authors contain imaginary numbers, based on a non existing back-radiation which exaggerates the influence of IR-sensitive gases.

The capability of mother earth to send back the heat she receives from the sun is much higher than would follow from such budgets.

CO_2 is not the reason for climate change. CO_2 is food for plants.

Governmental Policies based on such global and annual mean energy budgets, to tax the CO_2 emission, should be reconsidered.

Back-radiation follows from the two-stream Eddington equations of light propagation which tacitly has been assumed to be valid also for heat radiation. In this respect it is relevant, already indicated by Claes Johnson, to quote the statement by Sir Arthur Stanley Eddington himself in "Nature of the Physical World, 1915:

If someone points out that your pet theory of the universe is in disagreement with Maxwell's equations, then so much the worse for Maxwell's equations.
If it is found to be contradicted by observation, well, these experimentalists do bungle sometimes.
But if your theory is to found to be against the second law of thermodynamics, I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

Acknowledgement

The author is indepted to Joseph Postma, Nasif Nahle and Alan Siddons for their kindness to preview this paper and giving the author suggestions to improve it. The help of Hans Schreuder to edit and host this paper on his site and give it a broader distribution is appreciated. Bedankt Hans.

Appendix 1

For the one layer model equation (**5**) of the main text becomes with $\varepsilon_1=1$, $\varepsilon_2 = \varepsilon$ and $f_2= f$, and for a surface flux **q**:

$$\left[\begin{vmatrix} 1 & -f \\ 0 & \varepsilon \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ f & -f \end{vmatrix} \right] \begin{vmatrix} \theta_1 \\ \theta_2 \end{vmatrix} = \begin{vmatrix} q \\ 0 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} 1 & -f \\ -f & \varepsilon+f \end{vmatrix} \begin{vmatrix} \theta_1 \\ \theta_2 \end{vmatrix} = \begin{vmatrix} q \\ 0 \end{vmatrix}$$

The solution follows from Cramer's rule:

$$\begin{vmatrix} \theta_1 \\ \theta_2 \end{vmatrix} = \frac{1}{\varepsilon+f - f\hat{}2} \begin{vmatrix} \varepsilon+f & f \\ f & 1 \end{vmatrix} \begin{vmatrix} q \\ 0 \end{vmatrix}$$

$$\theta_1/q = (\varepsilon+f)/ (\varepsilon+f - f\hat{}2)$$

For $\varepsilon=f$, we find equation **(2)** from the main text:

$$\theta_1/q = 1/(1-f/2) \qquad \qquad AFT = (\theta_1/q)\hat{}0.25 = 1/(1-f/2)\hat{}0.25$$

AFT is the atmospheric factor indicating the relative temperature increase due to the semi-transparent atmosphere. It is sometimes called the green house factor, a misnomer since in the green houses of nurseries the temperature is higher than the environment not because of the fact that the glass roof hinders the IR-radiation, but because of the fact that convection of heat to the environment is hindered by the closed roof.

Appendix 2

Lagrange multipliers as an elegant technique for constraints to impose lapse rates.

The technique to use the system matrix in the non-classical reversed direction like in equation (**12**) in the main text, is a standard procedure in structural analysis by means of finite element techniques *e.g.* to impose variable boundary conditions and to define thereby varying reaction forces.
It is carried out by means of the technique of Lagrangian multipliers**.**
We can say that the system equation results from the minimization of a potential Π augmented with additional constraints.
In the case of a constraint of a temperature distribution, defined by the measured surface temperature and a measured lapse rate, the constraint is because of the "measured" θ_{LR} **:**

$\theta_i = \theta_{LRi}$ =====> **LAG*(θ-θLR)=0,** **LAG** =

$$LAG = \begin{vmatrix} 1 & 0 & . & . & 0 \\ 0 & 1 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & 1 \end{vmatrix}$$

In this case **LAG** is the identity matrix**,** because we put a constraint on all variables. The augmented potential **Π**, including the constraints, becomes:

$$\Pi = 1/2\ \theta' * K * \theta - \theta' * RHS + \lambda' * LAG * (\theta - \theta_{LR})$$

The parameters **λ** are called Lagrange multipliers.
The potential **Π** should have a stationery value for arbitrary variations of **δθ** and **δλ** of the components of **θ** and **λ**, from which follows:

$$\begin{bmatrix} K & LAG' \\ LAG & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \lambda \end{bmatrix} = \begin{bmatrix} RHS \\ LAG * \theta_{LR} \end{bmatrix}$$

For a **K** matrix of order 3**x3**, the augmented matrix is of order **6x6.**

$$\begin{vmatrix} k_{11} & k_{12} & k_{13} & 1 & 0 & 0 \\ k_{21} & k_{22} & k_{23} & 0 & 1 & 0 \\ k_{31} & k_{32} & k_{33} & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{vmatrix} = \begin{vmatrix} RHS_1 \\ RHS_2 \\ RHS_3 \\ \theta_{LR1} \\ \theta_{LR2} \\ \theta_{LR3} \end{vmatrix}$$

The solution of this system gives for the unknowns: $\theta = \theta_{LR}$.
This can be seen immediately by solving the last three equations.
Then we can write the first three equations as:

$$K * \theta_{LR} = RHS - \lambda$$

RHS is defined by the absorption of **qtoa:** the terms **RHSi** for **i=2,N** before **qtoa** hits the surface and the term **RHS1** for that part of **qtoa,** which has made it to the surface, but with a delay due to thermal inertia of the surface, be it ground or water.

The augmented right hand side, **RHS** - $\boldsymbol{\lambda}$, represents the necessary loading which can be handled by the mechanism of radiation with the temperature distribution according to the environmental lapse rate **ELR** and the surface temperature **Ts.**

In particular the components of the vector -$\boldsymbol{\lambda}$ represent heat deposits due to mechanisms, not yet taking into account by **RHS,** such as absorption of incoming sun light, deposit of sensible and latent heat by a convection mechanism, absorption and re-emission by solid particles due to volcanoes, etc. And not only transport of heat terms in vertical direction due to convection of sensible and latent heat, but also the effect of transverse transport by means of wind!

In the same way as the $\mathbf{q_{LR}}$ defined by equation (**14**) in the main text!

The Lagrange multipliers only serve to make the programming easier and more transparent. The reader who has difficulties with the technique of Lagrange multipliers can go back to (**14**) in the main text and the discussion about $\mathbf{q_{LR}}$, which is probably more intuitive.

Appendix 3

Figure 4 in the text is obtained by a MATLAB program using the formula below found in Wikipedia. [**13**]

The saturated adiabatic lapse rate is given approximately by this equation from the glossary of the American Meteorology Society:

$$\Gamma_w = g \, \frac{1 + \dfrac{H_v \, r}{R_{sd} \, T}}{c_{pd} + \dfrac{H_v^2 \, r \, \epsilon}{R_{sd} \, T^2}}$$

Whe
re

Γ_w	= Wet adiabatic lapse rate, K/m
g	= Earth's gravitational acceleration = 9.8076 m/s^2
H_v	= Heat of vaporization of water, J/kg
r	= The ratio of the mass of water vapor to the mass of dry air, kg/kg
R	= The universal gas constant = 8,314 J kmol^{-1} K^{-1}
M	= The molecular weight of any specific gas, kg/kmol = 28.964 for dry air and 18.015 for water vapor
R / M	= The specific gas constant of a gas, denoted as R_s
R_{sd}	= Specific gas constant of dry air = 287 J kg^{-1} K^{-1}
R_{sw}	= Specific gas constant of water vapor = 462 J kg^{-1} K^{-1}
ε	= The dimensionless ratio of the specific gas constant of dry air to the specific gas constant for water vapor = 0.6220

Appendix 4

Application of the model to predict the possibility of hot towers

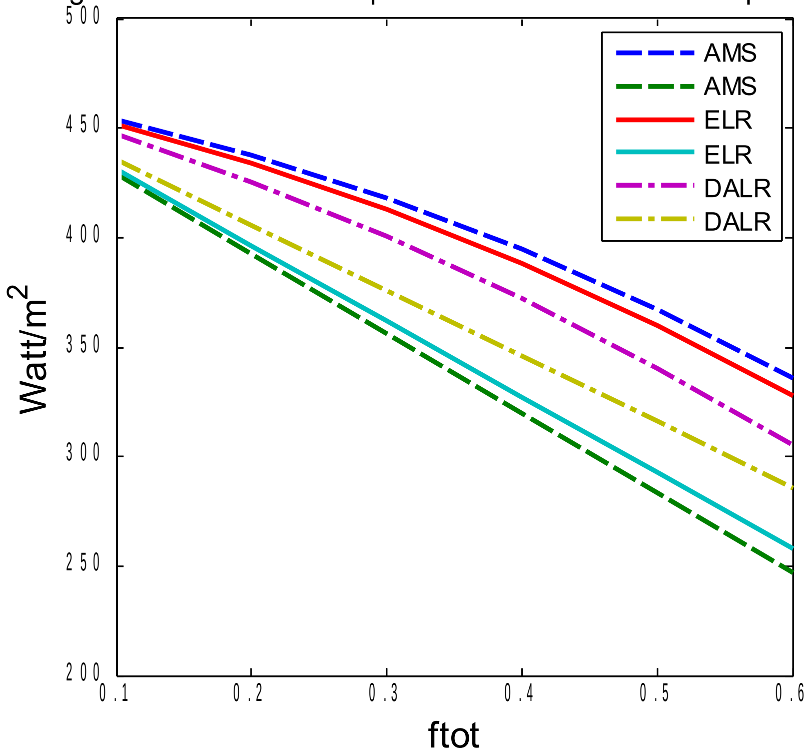
The model as presented in this paper is a fast tool to determine the amount of heat which can be evacuated by the mechanism of radiation in tropical zones. In figure **7** are given curves similar as in figure **4**, but for a surface temperature **T_s=28 C** and for the three lapse rates. Figure **7** shows that in tropical zones, and with shallow seas and a water temperature of **28 C** the atmosphere has a high humidity and as a consequence a higher absorption coefficient than the value **ftot= 0.2** for **2.5%** water vapor concentration.

The radiation mechanism might not be sufficient.

The **qtoa** should not be higher than the highest curve for the lapse rate for high moisture content according to AMS (see also fig **4** and appendix **3**).

The maximum **qtoa** which can be evacuated by radiation for an absorption coefficient for extreme humid air between **0.2** and **0.3** is of the order of **430** W/m^2**.**

Fig 7 OLRtot and OLRq for Ts=28 C and various lapse rates



The figure indicates that the radiation mechanism might not be sufficient. In figure **4**, the height of a hot tower is shown for **Ts =15 C** and relative humidity **0.85**, giving for these rather moderate conditions a height of **7.5** km.

See article by Joanne Simpson, the famous meteorologist and originator of the theory on hot towers. The article shows pictures of hot towers as high as 20 km. [**14**]

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